

---

# CHAPTER 3: Unified I-V Model

---

The development of separate model expressions for such device operation regimes as subthreshold and strong inversion were discussed in Chapter 2. Although these expressions can each accurately describe device behavior within their own respective region of operation, problems are likely to occur between two well-described regions or within transition regions. In order to circumvent this issue, a unified model should be synthesized to not only preserve region-specific expressions but also to ensure the continuities of current ( $I_{ds}$ ) and conductance ( $G_{\chi}$ ) and their derivatives in *all* transition regions as well. Such high standards are met in BSIM3v3. As a result, convergence and calculation efficiencies are much improved.

This chapter will describe the unified nature of BSIM3v3's model equations. While most of the parameter symbols in this chapter are explained in the following text, a complete description of all I-V model equation parameters can be found in the Appendix A.

## 3.1 Unified Channel Charge Density Expression

Separate expressions for channel charge density are shown below for subthreshold (Eq. (3.1.1a) and (3.1.1b)) and strong inversion (Eq. (3.1.2)). Both expressions are valid for small  $V_{ds}$ .

## Unified Channel Charge Density Expression

---

(3.1.1a)

$$Q_{chsub0} = Q_0 \exp\left(\frac{V_{gs} - V_{th}}{n v_t}\right)$$

where,  $Q_0$  is:

(3.1.1b)

$$Q_0 = \sqrt{\frac{q \epsilon_{si} N_{ch}}{2 \phi_s}} v_t \exp\left(-\frac{V_{off}}{n v_t}\right)$$

(3.1.2)

$$Q_{chs0} = C_{ox}(V_{gs} - V_{th})$$

In both Eqs. (3.1.1a) and (3.1.2), the parameters  $Q_{chsub0}$  and  $Q_{chs0}$  are the channel charge densities at the source for very small Vds. To form a unified expression, an effective (Vgs-Vth) function named Vgsteff is introduced to describe the channel charge characteristics from subthreshold to strong inversion:

(3.1.3)

$$V_{gsteff} = \frac{2 n v_t \ln \left[ 1 + \exp\left(\frac{V_{gs} - V_{th}}{2 n v_t}\right) \right]}{1 + 2 n C_{ox} \sqrt{\frac{2 \Phi_s}{q \epsilon_{si} N_{ch}}} \exp\left(-\frac{V_{gs} - V_{th} - 2V_{off}}{2 n v_t}\right)}$$

The unified channel charge density at the source end for both subthreshold and inversion region can therefore be written as:

(3.1.4)

$$Q_{chs0} = C_{ox} V_{gsteff}$$

Figures 3-1 and 3-2 show the "smoothness" of Eq. (3.1.4) from subthreshold to strong inversion regions. The Vgsteff expression will be used again in subsequent sections of this chapter to model the drain current.

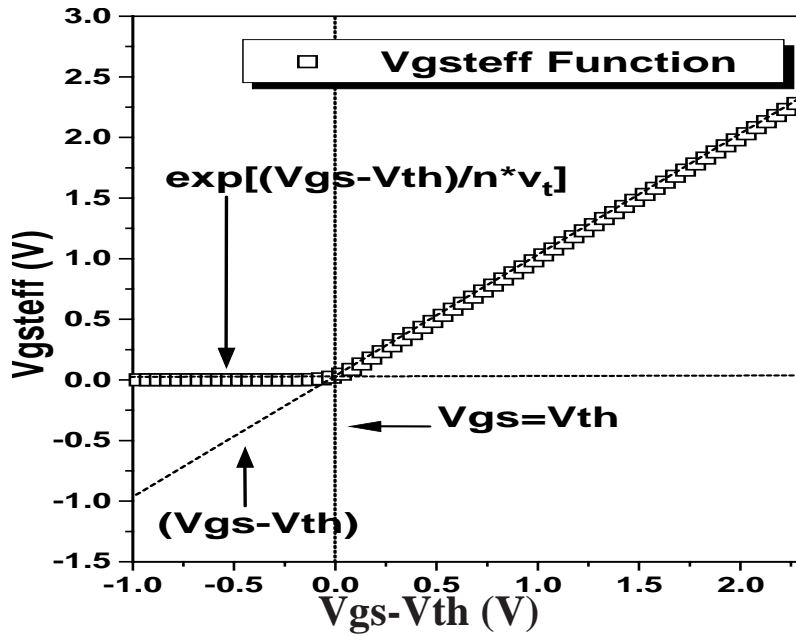


Figure 3-1. The Vgsteff function vs. (Vgs-Vth) in linear scale.

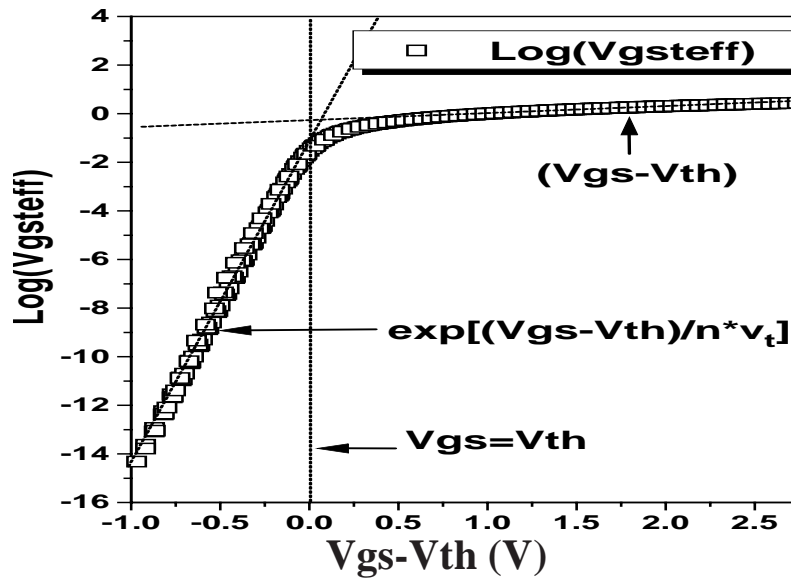


Figure 3-2. Vgsteff function vs. (Vgs-Vth) in log scale.

## Unified Channel Charge Density Expression

---

Eq. (3.1.4) serves as the cornerstone of the unified channel charge expression at the source for small  $V_{ds}$ . To account for the influence of  $V_{ds}$ , the  $V_{gsteff}$  function must keep track of the change in channel potential from the source to the drain. In other words, Eq. (3.1.4) will have to include a  $y$  dependence. To initiate this formulation, consider first the re-formulation of channel charge density for the case of **strong inversion**:

$$Q_{chs(y)} = C_{ox}(V_{gs} - V_{th} - A_{bulk}V_{F(y)}) \quad (3.1.5)$$

The parameter  $V_{F(y)}$  stands for the quasi-Fermi potential at any given point,  $y$ , along the channel with respect to the source. This equation can also be written as:

$$Q_{chs(y)} = Q_{chs0} + \Delta Q_{chs(y)} \quad (3.1.6)$$

The term  $\Delta Q_{chs(y)}$  is the incremental channel charge density induced by the drain voltage at point  $y$ . It can be expressed as:

$$\Delta Q_{chs(y)} = -C_{ox}A_{bulk}V_{F(y)} \quad (3.1.7)$$

For the **subthreshold region** ( $V_{gs} \ll V_{th}$ ), the channel charge density along the channel from source to drain can be written as:

## Unified Channel Charge Density Expression

---

$$\begin{aligned}
 Q_{chsubs(y)} &= Q_0 \exp\left(\frac{V_{gs} - V_{th} - A_{bulk} V_{F(y)}}{n v_t}\right) \\
 &= Q_{chsubs0} \exp\left(-\frac{A_{bulk} V_{F(y)}}{n v_t}\right)
 \end{aligned}
 \tag{3.1.8}$$

A Taylor series expansion of the right-hand side of Eq. (3.1.8) yields the following (keeping only the first two terms):

$$Q_{chsubs(y)} = Q_{chsubs0} \left(1 - \frac{A_{bulk} V_{F(y)}}{n v_t}\right)
 \tag{3.1.9}$$

Analogous to Eq. (3.1.6), Eq. (3.1.9) can also be written as:

$$Q_{chsubs(y)} = Q_{chsubs0} + \Delta Q_{chsubs(y)}
 \tag{3.1.10}$$

The parameter  $\Delta Q_{chsubs(y)}$  is the incremental channel charge density induced by the drain voltage in the subthreshold region. It can be written as:

$$\Delta Q_{chsubs(y)} = -\frac{A_{bulk} V_{F(y)}}{n v_t} Q_{chsubs0}
 \tag{3.1.11}$$

Note that Eq. (3.1.9) is valid only when  $V_{F(y)}$  is very small, which is maintained, fortunately, due to the fact that Eq. (3.1.9) is only used in the linear regime (i.e.  $V_{ds} \leq 2v_t$ ).

Eqs. (3.1.6) and (3.1.10) both have drain voltage dependencies. However, they are decouple and a unified expression for  $Q_{ch(y)}$  is desperately needed. To obtain a unified expression along the channel, we first assume:

## Unified Mobility Expression

---

(3.1.12)

$$\Delta Q_{ch(y)} = \frac{\Delta Q_{chs(y)} \Delta Q_{chsbs(y)}}{\Delta Q_{chs(y)} + \Delta Q_{chsbs(y)}}$$

Here,  $\Delta Q_{ch(y)}$  is the incremental channel charge density induced by the drain voltage. Substituting Eq. (3.1.7) and (3.1.11) into Eq. (3.1.12), we obtain:

(3.1.13)

$$\Delta Q_{ch(y)} = \frac{V_{F(y)}}{V_b} Q_{chs0}$$

where  $V_b = (V_{gsteff} + n \cdot v_t) / A_{bulk}$ . In order to remove any association between the variable  $n$  and bias dependencies ( $V_{gsteff}$ ) as well as to ensure more precise modeling of Eq. (3.1.8) for linear regimes (under subthreshold conditions), the variable  $n$  is replaced with 2. The expression for  $V_b$  now becomes:

(3.1.14)

$$V_b = \frac{V_{gsteff} + 2v_t}{A_{bulk}}$$

A unified expression for  $Q_{ch(y)}$  from subthreshold to strong inversion regimes is now at hand:

(3.1.15)

$$Q_{ch(y)} = Q_{chs0} \left(1 - \frac{V_{F(y)}}{V_b}\right)$$

The variable  $Q_{chs0}$  is given by Eq. (3.1.4).

## 3.2 Unified Mobility Expression

BSIM3v3 uses a unified mobility expression based on the  $V_{gsteff}$  expression of Eq. 3.1.3. Thus, we have:

## Unified Linear Current Expression

---

$$\text{(Mobmod=1)} \quad (3.2.1)$$

$$\mu_{eff} = \frac{\mu_o}{1 + (U_a + U_c V_{bseff}) \left( \frac{V_{gsteff} + 2V_{th}}{T_{OX}} \right) + U_b \left( \frac{V_{gsteff} + 2V_{th}}{T_{OX}} \right)^2}$$

To account for depletion mode devices, another mobility model option is given by the following:

$$\text{(Mobmod=2)} \quad (3.2.2)$$

$$\mu_{eff} = \frac{\mu_o}{1 + (U_a + U_c V_{bseff}) \left( \frac{V_{gsteff}}{T_{OX}} \right) + U_b \left( \frac{V_{gsteff}}{T_{OX}} \right)^2}$$

To consider the body bias dependence of Eq. 3.2.1 further, we have introduced the following expression:

$$\text{(For Mobmod=3)} \quad (3.2.3)$$

$$\mu_{eff} = \frac{\mu_o}{1 + \left[ U_a \left( \frac{V_{gsteff} + 2V_{th}}{T_{OX}} \right) + U_b \left( \frac{V_{gsteff} + 2V_{th}}{T_{OX}} \right)^2 \right] (1 + U_c V_{bseff})}$$

## 3.3 Unified Linear Current Expression

### 3.3.1 Intrinsic case (Rds=0)

Generally, the following expression [2] is used to account for both drift and diffusion current:

## Unified Linear Current Expression

---

$$I_{d(y)} = WQ_{ch(y)}\mu_{ne(y)}\frac{dV_{F(y)}}{dy} \quad (3.3.1)$$

where the parameter  $\mu_{ne(y)}$  can be written as:

$$\mu_{ne(y)} = \frac{\mu_{eff}}{1 + \frac{E_y}{E_{sat}}} \quad (3.3.2)$$

Substituting Eq. (3.3.2) in Eq. (3.3.1) we get:

$$I_{d(y)} = WQ_{chso}\left(1 - \frac{V_{F(y)}}{V_b}\right)\frac{\mu_{eff}}{1 + \frac{E_y}{E_{sat}}}\frac{dV_{F(y)}}{dy} \quad (3.3.3)$$

Eq. (3.3.3) resembles the equation used to model drain current in the strong inversion regime. However, it can now be used to describe the current characteristics in the subthreshold regime when  $V_{ds}$  is very small ( $V_{ds} < 2V_T$ ). Eq. (3.3.3) can now be integrated from the source to drain to get the expression for linear drain current in the channel. This expression is valid from the subthreshold regime to the strong inversion regime:

$$I_{ds0} = \frac{W\mu_{eff}Q_{chso}V_{ds}\left(1 - \frac{V_{ds}}{2V_b}\right)}{L\left(1 + \frac{V_{ds}}{V_a}\right)} \quad (3.3.4)$$

### 3.3.2 Extrinsic Case ( $R_{ds} > 0$ )

The current expression when  $R_{ds} > 0$  can be obtained based on Eq. (2.5.9) and Eq. (3.3.4). The expression for linear drain current from subthreshold to strong inversion is:

$$I_{ds} = \frac{I_{dso}}{1 + \frac{R_{ds} I_{dso}}{V_{ds}}} \quad (3.3.5)$$

Chapter 8 will illustrate the "smoothness" of this expression.

## 3.4 Unified Vdsat Expression

### 3.4.1 Intrinsic case ( $R_{ds}=0$ )

To get an expression for the electric field as a function of  $y$  along the channel, we integrate Eq. (3.3.1) from 0 to an arbitrary point  $y$ . The expression is as follows:

$$E_y = \frac{I_{dso}}{\sqrt{(WQ_{chs0}\mu_{eff} - \frac{I_{dso}}{E_{sat}})^2 - \frac{2I_{dso}WQ_{chs0}\mu_{eff}y}{V_b}}} \quad (3.4.1)$$

If we assume that drift velocity saturates when  $E_y = E_{sat}$ , we get the following expression for  $I_{dsat}$ :

## Unified Vdsat Expression

---

$$I_{dsat} = \frac{W\mu_{eff}Q_{chs0}E_{sat}LV_b}{2L(E_{sat}L + V_b)} \quad (3.4.2)$$

Let  $V_{ds}=V_{dsat}$  in Eq. (3.3.4) and set this equal to Eq. (3.4.2), we get the following expression for  $V_{dsat}$ :

$$V_{dsat} = \frac{E_{sat}L(V_{gsteff} + 2v_t)}{A_{bulk}E_{sat}L + V_{gsteff} + 2v_t} \quad (3.4.3)$$

### 3.4.2 Extrinsic Case ( $R_{ds}>0$ )

The  $V_{dsat}$  expression for the extrinsic case is formulated from Eq. (3.4.3) and Eq. (2.5.10) to be the following:

$$V_{dsat} = \frac{-b - \sqrt{b^2 - 4ac}}{2a} \quad (3.4.4a)$$

where,

$$a = A_{bulk}^2 W_{eff} v_{sat} C_{ox} R_{DS} + \left(\frac{1}{\lambda} - 1\right) A_{bulk} \quad (3.4.4b)$$

$$b = -\left( (V_{gsteff} + 2v_t) \left(\frac{2}{\lambda} - 1\right) + A_{bulk} E_{sat} L_{eff} + 3A_{bulk} (V_{gsteff} + 2v_t) W_{eff} v_{sat} C_{ox} R_{DS} \right) \quad (3.4.4c)$$

$$c = (V_{gsteff} + 2v_t) E_{sat} L_{eff} + 2(V_{gsteff} + 2v_t)^2 W_{eff} v_{sat} C_{ox} R_{DS} \quad (3.4.4d)$$

## Unified Saturation Current Expression

---

(3.4.4e)

$$\lambda = A_1 V_{gsteff} + A_2$$

The parameter  $\lambda$  is introduced to account for non-saturation effects. The parameters  $A_1$  and  $A_2$  are extracted.

### 3.5 Unified Saturation Current Expression

A unified expression for the saturation current from the subthreshold to the strong inversion regime can be formulated by introducing the  $V_{gsteff}$  function into Eq. (2.6.15). The resulting equations are the following:

(3.5.1)

$$I_{ds} = \frac{I_{dso}(V_{dsat})}{1 + \frac{R_{ds} I_{dso}(V_{dsat})}{V_{dsat}}} \left( 1 + \frac{V_{ds} - V_{dsat}}{V_A} \right) \left( 1 + \frac{V_{ds} - V_{dsat}}{V_{ASCBE}} \right)$$

where,

(3.5.2)

$$V_A = V_{Asat} + \left( 1 + \frac{P_{vag} V_{gsteff}}{E_{sat} L_{eff}} \right) \left( \frac{1}{V_{ACLM}} + \frac{1}{V_{ADIBLC}} \right)^{-1}$$

(3.5.3)

$$V_{Asat} = \frac{E_{sat} L_{eff} + V_{dsat} + 2 R_{DS} V_{sat} C_{ox} W_{eff} V_{gsteff} \left[ 1 - \frac{A_{bulk} V_{dsat}}{2(V_{gsteff} + 2V_t)} \right]}{2 / \lambda - 1 + R_{DS} V_{sat} C_{ox} W_{eff} A_{bulk}}$$

(3.5.4)

$$V_{ACLM} = \frac{A_{bulk} E_{sat} L_{eff} + V_{gsteff}}{P_{CLM} A_{bulk} E_{sat} l_{itl}} (V_{ds} - V_{dsat})$$

## Single Current Expression for All Operational Regimes of Vgs and Vds

---

$$V_{ADIBLC} = \frac{(V_{gsteff} + 2V_t)}{\theta_{rout}(1 + P_{DIBLCB}V_{bseff})} \left( 1 - \frac{A_{bulk}V_{dsat}}{A_{bulk}V_{dsat} + V_{gsteff} + 2V_t} \right) \quad (3.5.5)$$

$$\theta_{rout} = P_{DIBLC1} \left[ \exp\left(-D_{ROUT} \frac{L_{eff}}{2l_{t0}}\right) + 2 \exp\left(-D_{ROUT} \frac{L_{eff}}{l_{t0}}\right) \right] + P_{DIBLC2} \quad (3.5.6)$$

$$\frac{1}{V_{ASCBE}} = \frac{P_{sobe2}}{L_{eff}} \exp\left(\frac{-P_{sobe1} l_{t0}}{V_{ds} - V_{dsat}}\right) \quad (3.5.7)$$

## 3.6 Single Current Expression for All Operational Regimes of Vgs and Vds

The  $V_{gsteff}$  function introduced in Chapter 2 gave a unified expression for the linear drain current from subthreshold to strong inversion as well as for the saturation drain current from subthreshold to strong inversion, *separately*. In order to link the continuous linear current with that of the continuous saturation current, a smooth function for  $V_{ds}$  is introduced. In the past, several smooth functions have been proposed for MOSFET modeling [22-24]. For BSIM3v3, the smooth function used is similar to that proposed by R. M. D. A. Velghe et al [24]. The overall current equation for both linear and saturation current now becomes:

$$I_{ds} = \frac{I_{dso}(V_{dseff})}{1 + \frac{R_{ds}I_{dso}(V_{dseff})}{V_{dseff}}} \left( 1 + \frac{V_{ds} - V_{dseff}}{V_A} \right) \left( 1 + \frac{V_{ds} - V_{dseff}}{V_{ASCBE}} \right) \quad (3.6.1)$$

Most of the previous equations which contain  $V_{ds}$  and  $V_{dsat}$  dependencies are now substituted with the  $V_{dseff}$  function. For example, Eq. (3.5.4) now becomes:

## Single Current Expression for All Operational Regimes of Vgs and Vds

---

$$V_{ACLM} = \frac{A_{bulk} E_{sat} L_{eff} + V_{gsteff}}{P_{CLM} A_{bulk} E_{sat} litl} (V_{ds} - V_{dseff}) \quad (3.6.2)$$

Similarly, Eq. (3.5.7) now becomes:

$$\frac{1}{V_{ASCBE}} = \frac{P_{scbe2}}{L_{eff}} \exp\left(\frac{-P_{scbe1} litl}{V_{ds} - V_{dseff}}\right) \quad (3.6.3)$$

The Vdseff expression is written as:

$$V_{dseff} = V_{dsat} - \frac{1}{2} \left( V_{dsat} - V_{ds} - \delta + \sqrt{(V_{dsat} - V_{ds} - \delta)^2 + 4\delta V_{dsat}} \right) \quad (3.6.4)$$

The expression for Vdsat is that given under Section 3.4. The parameter  $\delta$  is an extracted constant. The dependence of Vdseff on Vds is given in Figure 3-3. The Vdseff function follows Vds in the linear region and tends to Vdsat in the saturation region. Figure 3-4 shows the effect of  $\delta$  on the transition region between linear and saturation regimes.

## Single Current Expression for All Operational Regimes of $V_{gs}$ and $V_{ds}$

---

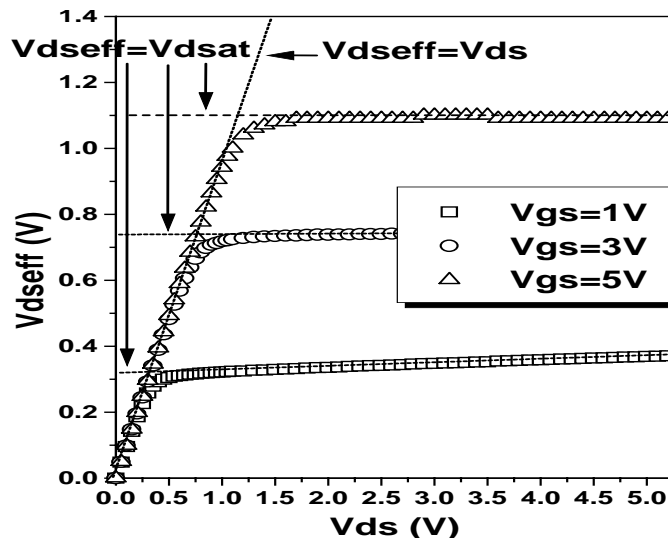


Figure 3-3.  $V_{dseff}$  vs.  $V_{ds}$  for  $\delta=0.01$  and different  $V_{gs}$ .

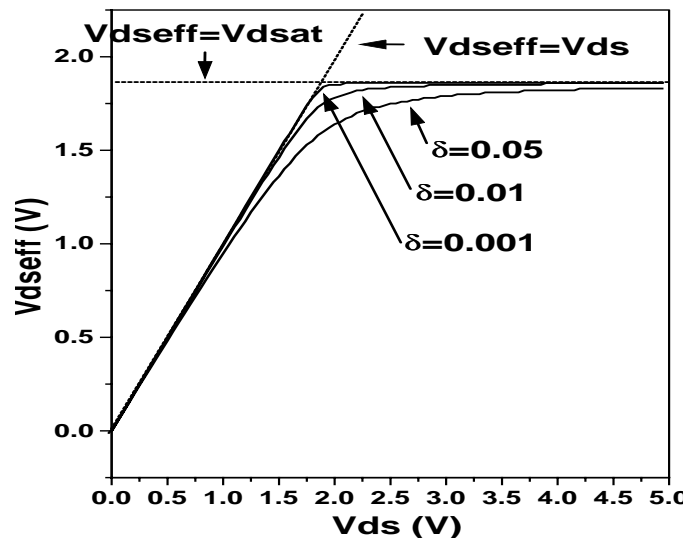


Figure 3-4.  $V_{dseff}$  vs.  $V_{ds}$  for  $V_{gs}=3V$  and different  $\delta$  values.

### 3.7 Substrate Current

BSIMv3 uses the Eq. (3.7.1) to model substrate current.

$$I_{sub} = \frac{\alpha_0}{L_{eff}} (V_{ds} - V_{dseff}) \exp\left(-\frac{\beta_0}{V_{ds} - V_{dseff}}\right) \frac{I_{dso}}{1 + \frac{R_{ds} I_{dso}}{V_{dseff}}} \left(1 + \frac{V_{ds} - V_{dseff}}{V_A}\right) \quad (3.7.1)$$

The parameters  $\alpha_0$  and  $\beta_0$  refer to impact ionization current.

### 3.8 A Note on Vbs

In BSIM3v3, all  $V_{bs}$  terms which have appeared in Chapters 2 and 3 have been substituted with a  $V_{bseff}$  expression as shown in Eq. (3.8.1). This is done in order to set an upper bound for the body bias value during simulations. Unreasonable values can occur if this expression is not introduced.

$$V_{bseff} = V_{bc} + 0.5[V_{bs} - V_{bc} - \delta_1 + \sqrt{(V_{bs} - V_{bc} - \delta_1)^2 - 4\delta_1 V_{bc}}] \quad (3.8.1)$$

where  $\delta_1 = 0.001$ .

The parameter  $V_{bc}$  is the maximum allowable  $V_{bs}$  value and is obtained based on the condition of  $dV_{th}/dV_{bs} = 0$  for the  $V_{th}$  expression of 2.1.4.

